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Modeling of Lightning Incidence to Tall Structures
Part I: Theory

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Abstract – The paper generalizes a recent physical approach to assess negative downward lightning incidence to apply to tall masts and hilly regions.

Criteria for occurrence of an upward flash from a tall structure under negative cloud are formulated, both for flat and hilly terrain.

Finally the effect of structure on statistical stroke current distribution is analytically investigated.

Extensive computer investigation to apply the theory as well as comparison with field observations are reported in a companion paper.

Keywords: Lightning, Leader, Mast, Shielding.

INTRODUCTION

Considerable research effort has already gone into assessment of lightning attractiveness of structures and conductors to direct lightning strokes, as recently reviewed by CIGRE [1] and IEEE [2] Working Groups on Lightning Performance of Transmission Lines. For understandable reasons, apart from empirical analysis, modeling of lightning incidence dealt exclusively with the downward negative flash, moderate power structure heights and mostly with flat terrain [3]-[5].

Tall structures such as power line towers at river crossings and telecommunication masts are frequently hit by lightning [6] and several tall free-standing structures are being monitored by the power industry for lightning data collection [7].

It has been noted that stroke incidences to structures of moderate heights in hilly and mountainous regions, presumably caused mostly by downward negative flash, considerably exceed those of similar structures on flat ground [8].

It has also been reported that as structure height exceeds roughly 100 m an increasing number of lightning incidence is caused by upward flash without any visible downward leader activity [3], [6]. On the other hand our knowledge of the probability of an upward flash is rather sketchy and a shielding theory for upward lightning has just recently been attempted [9].

Another controversial point is that of the effect of structure height on the return stroke current distribution [10]. Using the electrogeometric model [11] and a modified version thereof [12], conflicting results on structure height effects were reached and significantly different median currents of strokes to "open ground" were obtained.

In view of the above situation, the present paper is prepared with the following objectives:

- to generalize the physical approach of ref. [4] to account for the effects of hilly terrain on negative downward direct stroke incidence to tall free-standing structures;
- to formulate criteria for the occurrence of an upward flash caused by negative cloud without significant downward negative leader activity, both on flat and hilly terrains;
- to develop a rigorous mathematical approach to assess the effect of structure height on downward negative stroke current distribution and establish the correlations between such distribution and those for horizontal conductors and "open ground".

An extensive computer investigation of the effect of different parameters on stroke incidence as well as comparison with field observation is included in a companion paper.

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THEORY

Downward Negative Flash

Basic Approach

A basic approach for determining incidence, due to a conventional downward negative lightning, to structures and conductors of moderate heights, normally of interest to electric utilities, is described in ref. [4]. The following stages of a lightning stroke incidence were considered:

- as the downward negative leader approaches ground, it induces a transient electric field in the vicinity of a free standing structure maintained at ground potential;
- depending on the negative leader charge (return stroke current), distance from the structure as well as the structure geometry, corona streamers may be initiated from the structure;
- under appropriate conditions formulated in ref. [4], a streamer-positive leader transition takes place, with the positive leader propagating in the electric field seeking an encounter with the descending negative leader;
- under appropriate conditions described in ref. [4], an encounter between the two leaders may indeed take place, when the gap between the two leader tips is bridged by positive and negative streamers, through what is termed a final jump.

For a descending negative leader at given radial (for structures) or lateral (for conductors) distance, return stroke current and structure geometry, the computation comprises the following elements:

- determine the induced potential, due to the negative leader charge, at the structure tip position to decide whether positive leader inception is feasible, and if so, calculate the height of the negative leader at inception;
- following positive leader inception, trace leader trajectory and determine whether an encounter with negative leader is attainable;
- through an iterative process, determine the maximum distance of the leader from the structure, for which an encounter is possible *i.e.* the attractive radius (structure) or lateral attractive distance (conductor).

Model Generalization

Figure 1 shows a schematic diagram of free-standing structures on flat ground and on a mountain top. For analytic solutions the mountain shape has to be simplified and is here represented as a semi-ellipsoid with semi-minor axis a and semi-major axis

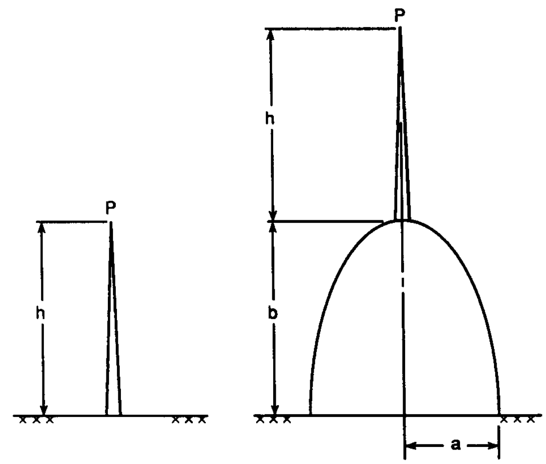

Fig. 1 Schematic diagram of free-standing mast of height: h on flat ground and over a mountain simulated as semi-ellipsoid (prolate spheroid) with semiminor axis (base radius): a, and semimajor axis (height): b.

(height) b. A special case considered is that of a hemisphere of radius a.

As in ref. [4], it is assumed that the total charge of the negative descending leader is only a function of the return stroke current. The charge distribution is such that it decays linearly from its highest value at the structure top level to reach zero at the base of the cloud. Calculation of the negative-leader-induced potential U_{il} at the structure tip position on mountain top can be made with charge simulation, taking into account the leader charge as well as image charges on both the ground plane and the mountain surface. In ref. [4], the effect of ambient ground field E_g due to cloud charges, as distinct from that of the negative leader, was neglected which as will be shown below, is a good approximation for structures of moderate height on flat terrain. For tall structures and mountainous regions treated in this paper, the effect of potential induced due to cloud charges and other distant space charges should be added to that induced due to the negative descending leader. The total induced potential U_i will then be:

$$U_i = \int_o^h E_g(z)\, dz + U_{il} \tag{1}$$

For a structure on flat ground it will be assumed that E_g will be sensibly independent of height:

$$U_i = E_g h + U_{il} \tag{2}$$

In the above it is implicitly assumed, that while the total field in the vicinity of the structure will increase considerably during the negative leader descent, the background field E_g will remain practically constant.

The critical value U_{lc} necessary for continuous positive leader inception and propagation can be obtained by calculating, using charge simulation technique, the function R needed for the expression [13]:

$$U_{lc} = 1556/[1 + \frac{7.78}{R}] \qquad (\text{kV}, \text{m}) \qquad (3)$$

In mountainous regions, the effect of reduced relative air density δ can be introduced in (3) as given in [14]:

$$U_{lc} = 1556/[1 + \frac{7.78}{\delta R}] \qquad (\text{kV}, \text{m}) \qquad (4)$$

It should be noted however that for tall structures $R \gg 7.78$ m, so that the effect of reduced air density on U_{lc} will normally be negligible.

Table 1 gives some numerical examples of the function R, obtained by charge simulation, for different structure heights h and different dimensions of the semi-ellipsoid-shaped mountain. Note that for flat ground R/h would be equal to 2, so that for a structure with $h = 60$ m the semi-ellipsoid with $a = 3000$ m and $b = 300$ m practically represents flat ground, as far as the critical positive leader inception voltage is concerned.

TABLE 1
Values of R Calculated by Charge Simulation Ellipsoid Mountain Shape

h, m	$a, m/b, m$	R, m	$R/2h$
40	100/300	122	1.53
60	100/300	192	1.60
80	100/300	275	1.72
100	100/300	364	1.82
150	100/300	598	1.99
60	3000/300	125	1.04

For the case of a hemispherical shaped mountain of radius a and structure of height h at the top, an analytical expression of the function R can be obtained as:

$$R = 2(h+a)/\left[1 + \frac{2a(h+a)}{(h+a)^2 - a^2} - \frac{2a(h+a)}{(h+a)^2 + a^2}\right] \qquad (5)$$

As shown in Fig. 2, for both $a/h \ll 1$ (flat ground) and $a/h \gg 1$ (big plateau), $R \rightarrow 2h$ as expected.

From Fig. 2, for $h = 40$ m and $a = 300$ m, one obtains $R = 85$ m, compared with 122 m for the more steep semi-ellipsoid of the same height in Table 1. While the difference in R is significant, the impact on U_{lc}, expression (4) above, is minimal since for the hemispherical mountain $U_{lc} = 1422$ kV while for the semi-ellipsoidal $U_{lc} = 1460$ kV, for the mast height under consideration.

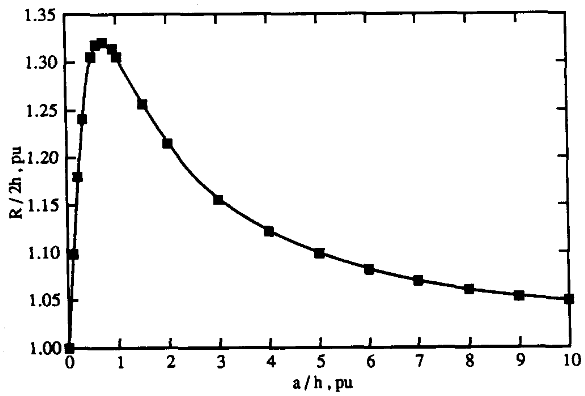

Fig. 2 Variation of the function R, normalized to a base $2h$, with the ratio a/h for a mast of height h at the top of a hemispherical mountain of height a.

In deciding the final jump, it is necessary to calculate the potential difference between the tips of the negative descending and the positive ascending leaders. The magnitude of the positive leader tip potential is the same as the positive leader voltage drop corresponding to the length involved. In ref. [4] that voltage drop was taken from data applicable to positive switching impulses. That approach represents a good approximation for the power line structure heights of ref. [4] where, at the final jump, the positive leader voltage drop is generally quite small compared to the negative descending leader tip potential. For tall structures and mountainous region, where much longer positive leaders may be involved a better approximation is needed.

In general, for a positive leader of length ℓ, the leader voltage drop ΔU_ℓ can be expressed as [15]:

$$\Delta U_\ell = \ell . E_\infty + x_o E_\infty \ln\left(\frac{E_i}{E_\infty} - \frac{E_i - E_\infty}{E_\infty} \exp(-\ell/x_o)\right) \qquad (6)$$

where

E_i is the minimum positive streamer gradient $\simeq 400$ kV/m

E_∞ is the final quasi-stationary leader gradient
$x_o = v\Theta$
Θ is the arc (leader) time constant
v is the ascending positive leader speed.

For laboratory gaps with positive leader current around 1A, E_∞ is generally taken in the range 30-50 kV/m [15], while v is 1-2x10^4 m/s. For the positive ascending leader we are dealing with currents around 100A [16] and speed of the order of 10^5 m/s [7]. At that current level the stationary gradient of a free burning arc in air is about 1 kV/m [17]. Furthermore, Bazelyan suggested [18] that the positive leader gradient is inversely proportional to the square root of the current. This leads us to a value of 3 kV/m as a reasonable estimate for E_∞ at the 100A level under consideration. Berger [16] suggested values in the range 2-6 kV/m.

For a free-burning stationary arc in the laboratory, the author's own measurements [19] indicate that for a current of 100A, the arc time-constant will be approximately 100 μs. However with view of the leader movement and cooling due to relatively strong wind normally associated with lightning, we will retain $\Theta = 50\mu$s, adopted in ref. [15].

With the above parameters, $x_o = 5$m and since we are normally dealing with positive leader lengths $\ell \gg x_o$, expression (5) reduces to:

$$\Delta U_\ell = \ell E_\infty + x_o E_\infty \ln \frac{E_i}{E_\infty} \qquad (7)$$

A numerical study showed that for a 40 m mast on flat ground and a return stroke current of 31 kA, variation of E_∞ from 3 kV/m to 30 kV/m resulted in a 7% reduction in the attractive radius. The corresponding reduction for an 80 m mast on a 600 m high mountain amounted to 15%.

At the final jump, with the inter-leader-tip gap bridged by positive and negative streamers, the mean streamer gradient E_s has been taken [4] as 500 kV/m. Here, the effect of reduced air density at high altitudes is introduced:

$$E_s = 500\delta \qquad \text{(kV/m)} \qquad (8)$$

where δ is here calculated at the height of the final jump, which would tend to increase the attractive radius, for otherwise the same conditions.

Upward Flash

The situation analyzed here is that of free-standing tall mast in flat or mountain region under a negatively charged cloud. The objective is to develop necessary and sufficient criteria for inception and continuous propagation towards the cloud of a positive leader emanating from the structure, without any significant descending leader.

In laboratory experiments, where long air gaps are exposed to high direct voltage, breakdown normally takes place by a streamer mechanism without significant leader formation. This is thought to be due to space charge effects, which render the leader inception criterion unsatisfied before the gap is completely bridged by streamers [20]. However basic differences exist between conventional dielectric direct voltage tests on laboratory air gaps and a free-standing structure under negative cloud:

1. Laboratory gaps are at most a few meters long and are normally tested in a relatively confined space, which considerably enhances the influence of space charge. On the other hand in lightning we are dealing with distances in the range of several hundred meters to a few kilometers, in open surroundings, with often strong wind and rain associated with lightning activities.

2. Laboratory tests are normally carried out with constant stress. This is not the case for the lightning situation under investigation, where E_g will in general vary with time due to cloud motion with associated charge drift, dispersion of space charge pockets by rain and wind, lightning activity within the cloud etc.

In ref. [9] the events leading to an upward flash were suggested as follows:

1. A thundercloud of relatively low height approaches from a certain direction.

2. The electric field at the top of a grounded structure increases by the charges in the thundercloud.

3. When the electric field exceeds a critical value, an upward leader begins to develop from the top of the structure towards the charge center of the thundercloud.

4. When the upward leader reaches the charge center, lightning occurs.

As will be seen below, the present theory is in basic agreement with this description.

The leader inception criterion [13] does take into account the effect of positive space charge due to stramers emanating from the structure. Furthermore, the effect of any distant space charge pockets is implicitly included in the value of E_g.

The basic difference between the present situation of upward flash and conventional downward negative lightning is that the induced potential at the position of structure top will be essentially determined by the ground field:

$$U_i = \int_o^h E_g(z)\, dz \tag{9}$$

which follows from (1) by neglecting the term U_{il}.

For positive leader inception:

$$U_i \geq U_{lc} \tag{10}$$

where U_{lc} is given by expression (4).

Leader inception will accordingly take place when the ground field reaches the critical level necessary to satisfy the above criterion, which incidentally is not at all restricted to slowly varying fields [13].

For conventional lightning, with a descending negative leader, the stress at the positive leader tip is normally enhanced as it progresses along its trajectory. This is not necessarily the case for the upward flash since positive leader tip potential is progressively reduced by the leader voltage drop while cloud base potential remains unchanged. This means that in general, (10) is a necessary but not sufficient criterion. A necessary and sufficient criterion will emerge if we impose that at any point during ascent the condition for continuous leader inception and propagation is satisfied.

The induced voltage (magnitude) at any point z will be designated:

$$U_i(z) = \textstyle\int_o^z E_g(z)\, dz \tag{9a}$$

and the critical inception voltage $U_c(z)$:

$$U_c(z) = 1556/[1 + 7.78/\delta \cdot R(z)] \tag{11}$$

Assuming for simplicity that the positive leader is ascending vertically, at a length ℓ_z the leader voltage drop will be:

$$\Delta U_\ell(z) = \ell_z E_\infty + x_o E_\infty \ln \frac{E_i}{E_\infty} \tag{12}$$

The general criterion will then be:

$$U_i(z) \geq U_c(z) + \Delta U_\ell(z) \tag{13}$$

with $U_i(z), U_c(z)$ and $\Delta U_\ell(z)$ given by (9a), (11), (12) respectively. $U_i(z)$ could be expressed as:

$$U_i(z) = \int_o^h E_g(z)\, dz + \int_o^{\ell_z} E_g(\ell) d\ell \tag{14}$$

Since we are dealing with tall structures, the variation of $U_c(z)$ with z can be neglected, so that:

$$U_c(z) = U_c = 1556/[1 + 7.78/\delta \cdot R] \tag{15}$$

Substituting in (13)

$$\int_o^h E_g(z) dz + \int_o^{\ell_z} E_g(\ell) d\ell \;\geq\; 1556/[1 + 7.78/\delta \cdot R] + \ell_z E_\infty + x_o E_\infty \ln \frac{E_i}{E_\infty} \tag{16}$$

The case of flat terrain, with E_g independent of height, is very instructive, for (16) then becomes:

$$E_g h \;\geq\; 1556/[1 + 7.78/\delta R] \;+\; x_o E_\infty \ln \frac{E_i}{E_\infty} + \ell_z [E_\infty - E_g] \tag{17}$$

Since this criterion has to be fulfilled for any value of ℓ_z, this can be satisfied if and only if:

$$E_g h \;\geq\; 1556/[1 + 7.78/\delta R] \;+\; x_o E_\infty \ln \frac{E_i}{E_\infty} \tag{18a}$$

and

$$E_g \;\geq\; E_\infty \tag{18b}$$

which constitute the necessary and sufficient criteria for the occurrence of the upward flash for flat terrain.

For the general case of mountainous region, the corresponding criteria for occurrence of the upward flash become:

$$\int_o^h E_g(z) dz \;\geq 1556/[1 + 7.78/\delta \cdot R] \;+\; x_o E_\infty \ln \frac{E_i}{E_\infty} \tag{19a}$$

and

$$\frac{1}{\ell_z} \int_o^{\ell_z} E_g(\ell)\, d\ell \;\geq\; E_\infty \tag{19b}$$

The L.H.S. is the mean gradient $\overline{E}_g$ of the ground field along the leader path, so that the second criterion can be written as:

$$\overline{E}_g \;\geq\; E_\infty \tag{19b}$$

Since R increases with structure height, criterion (19a), for $E_g(z) > 0$, can always be satisfied by a structure of sufficient height. On the other hand criterion (19b) states that there is a minimum ground field below which no upward flash will occur whatever the height of the structure might be. Criterion (19b) can be particularly important when extremely large heights are involved *e.g.* with conducting wire connected balloons, or if we stretch the application, rocket trigged lightning. Since (19b) has to be satisfied for any length of the positive leader, this will practically mean that (19b) and (18b) will be the same *i.e.* the background uniform field E_g, without considering local enhancement at the mountain top, has to exceed E_∞ if ever a complete upward flash is to become possible.

For a conducting ellipsoid as shown in Fig. 1b, immersed in a uniform electric field E_g, the potential induced across the height h, given by the L.H.S. of (19a) can be expressed as [21]:

$$\begin{aligned} U_i &= \int_o^h E_g(z)\, dz \\ &= E_g \cdot (h+b)\left\{1 - \left[coth^{-1}((1+h/b)\Theta) - (\Theta(1+\frac{h}{b}))^{-1}\right] / [coth^{-1}\Theta - 1/\Theta]\right\} \end{aligned} \quad (20)$$

where $\Theta = b/\sqrt{b^2 - a^2}$, which is valid for $b \geq a$. The dependence of U_i/hE_g on b/h for several values of a/b is illustrated in Fig. 3.

For a hemisphere of radius a, the induced potential simplifies to:

$$U_i = \int_o^h E_g(z)\, dz = E_g \cdot (h+a)[1 - a^3/(h+a)^3] \quad (21)$$

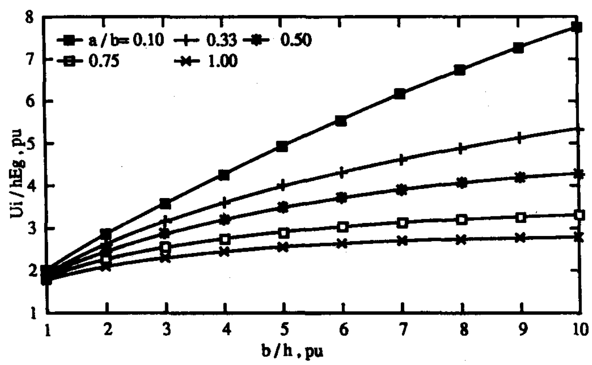

Fig. 3 Variation of the induced voltage U_i, due to an ambient ground field E_g, over a mast of height h, as function of the ratio between the ellipsoid mountain and mast heights.

Strike Assessment

For the conventional negative downward flash, the number of strikes per year N_d is related to the ground flash density N_g and the overall attractive radius R_{ao} [4] by:

$$N_d = \pi R_{ao}^2\, N_g \quad (22)$$

where $R_{ao} = R_{ao}(h)$

The number of upward flashes per year on the other hand is determined by the number of times, at the location of the structure, the criteria for upward flash are satisfied *i.e.* the critical ground field E_{gc} is exceeded. If in spite of the charge released in a flash, of some tens to a few hundred coulombs [16], the ground field at the same structure location, within the same storm, again exceeds E_{gc}, another upward flash will occur. This phenomenon however appears unlikely to be repeated frequently.

The number of upward flashes per year N_u could then be expressed as the product of the number of storms per year T_d and the cumulative probability, that once a storm did occur, the critical ground field E_{gc} is exceeded *i.e.*

$$N_u = T_d \cdot P(E_g > E_{gc}) \quad (23)$$

To derive an estimate of the probability distribution of the time-varying random variable E_g, available data on ground field-duration characteristics under thunder storm conditions will be used. Analysis of such data [22], [23] suggests that the cumulative probability function $P(E_g > E_{gc})$ could be approximated by an exponential function:

$$P(E_g > E_{gc}) = \exp(-(E_{gc} - E_{go})/E_{g1}) \quad (24)$$

for $E_{gc} > E_{go}$, where E_{go} is a threshold field, once a storm did occur and E_{g1} is a shape parameter. Fig. 4 shows that Simpson's ground field measurements [22], for $E_g > 2$ kV/m under disturbed weather (shower) conditions, can be well represented by expression (24) with $E_{go} = 2$kV/m and $E_{g1} = 2$ kV/m. Simpson stated that it was impossible to draw a hard and fast line between showers and thunderstorms [22]. We will call this distribution: 1 and suggest that it reasonably covers ordinary situations where it has often been reported that the ground field rarely exceeds 10 kV/m under lightning storm conditions. Fig. 5 on the other hand shows that Eriksson's measurements [23] obtained on a plateau of altitude 1400 m in South Africa, are better represented by expression (24) with $E_{go} = 3$ kV/m and $E_{g1} = 6$ kV/m. This distribution

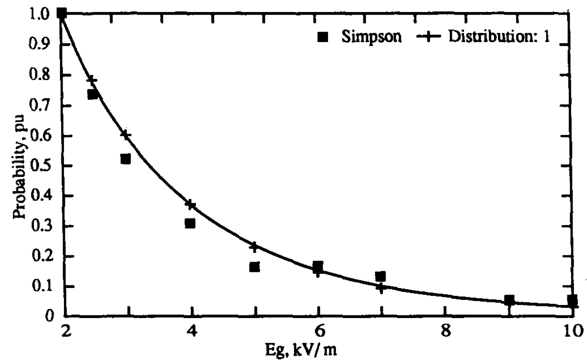

Fig. 4 Exponential ground field distribution 1 compared to Simpson's data [22].

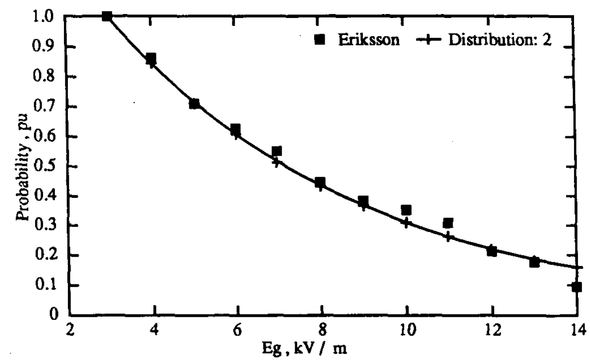

Fig. 5 Comparison of exponential ground field distribution 2 with Eriksson's data [23].

also appears to cover the range of ground fields reported from winter lightning in Japan [24] and will be referred to as distribution: 2. Both these distributions, which should be looked at as typical examples, will be used in the numerical investigation of a companion paper.

Substituting from (24) in (23):

$$N_u = T_d \cdot \exp(-(E_{gc} - E_{go})/E_{g1}) \qquad (25)$$

The probability P_u of an upward flash, will then be calculated from N_d and N_u given respectively by (22) and (25):

$$P_u = N_u/(N_u + N_d) \qquad (26)$$

Effect of Structure on Stroke Current

Let the probability density functions of stroke currents due to negative downward flash, for a mast and a conductor each of height h and to "open ground" be in general: $f_s(I,h)$, $f_c(I,h)$ and $f_g(I)$ respectively. Let the corresponding cumulative probability functions for current I to be exceeded be: $P_s(I,h), P_c(I,h)$ and $P_g(I)$.

Let $r(h,I)$ be the attractive radius of a mast of height h at a stroke current I. Let also $d(h,I)$ be the lateral attractive distance of a horizontal conductor of height h for the same current I.

The relationship between f_s and f_g will be:

$$f_s(I,h) = \frac{r^2(h,I)\, f_g(I)}{\int_o^\infty r^2(h,I)\, f_g(I)\, dI} \qquad (27)$$

which would permit the determination of the probability density function $f_s(I,h)$ for a mast, if the probability density function for "open ground" $f_s(I)$ was known. Simple manipulation leads to the relationship:

$$f_g(I) = \frac{f_s(I,h)}{r^2(h,I) \int_o^\infty f_s(I,h)/r^2(h,I)\, dI} \qquad (28)$$

which would permit the determination of $f_g(I)$ from $f_s(I,h)$.

Similarly for a horizontal conductor:

$$f_c(I,h) = \frac{d(h,I)\, f_g(I)}{\int_o^\infty d(h,I)\, f_g(I)\, dI} \qquad (29)$$

and corresponding to (28):

$$f_g(I) = \frac{f_c(I,h)}{d(h,I) \int_o^\infty f_c(I,h)/d(h,I)\, dI} \qquad (30)$$

The relationship between f_s and f_c can be obtained by eliminating $f_g(I)$ between (30) and (27):

$$f_s(I,h) = \frac{r^2(h,I)\, f_c(I,h)}{d(h,I) \int_o^\infty r^2(h,I) \cdot f_c(I,h)/d(h,I)\, dI} \qquad (31)$$

From which:

$$P_s(I_o,h) = \frac{\int_{I_o}^\infty r^2(h,I) f_c(I,h)/d(h,I)\, dI}{\int_o^\infty r^2(h,I) f_c(I,h)/d(h,I)\, dI} \qquad (32)$$

which permits the determination of P_s from knowledge of f_c. Similarly

$$P_c(I_o,h) = \frac{\int_{I_o}^\infty d(h,I) f_s(I,h)/r^2(h,I)\, dI}{\int_o^\infty d(h,I) f_s(I,h)/r^2(h,I)\, dI} \qquad (33)$$

Already some general conclusions can be drawn from the above analysis. If $r(h,I)$ can be expressed as a product of two explicit functions in h and I *i.e.*

$$r(h,I) = r_1(h) \cdot r_2(I) \tag{34}$$

a special case of which is:

$$r(h,I) = A_s h^{\alpha_s} I^{\beta_s} \tag{35}$$

with A_s, α_s and β_s as constants independent of I or h, which was previously found to constitute, within certain limits of h and I, a good approximation of modelling results [3], [4], then (27) immediately shows that:

$$f_s(I,h) = \frac{r_2^2(I) f_g(I)}{\int_o^\infty r_2^2(I) f_g(I)\, dI} = f_s(I) \tag{36}$$

which means that, within the range of validity of the above approximation, the stroke current distribution is independent of structure height.

Similarly if $d(h,I)$ can be expressed as:

$$d(h,I) = d_1(h) \cdot d_2(I) \tag{37}$$

of which the special case

$$d(h,I) = A_c h^{\alpha_c} I^{\beta_c} \tag{38}$$

with A_c, α_c and β_c as constants independent of h or I, was previously found, for practical conductor heights, to constitute a good approximation of model results [4], then (29) immediately shows that:

$$f_c(I,h) = \frac{d_2(I) f_g(I)}{\int_o^\infty d_2(I) f_g(I)\, dI} = f_c(I) \tag{39}$$

which again means that within the limits of the above approximation, the stroke current distribution is independent of conductor height.

Since $r^2(h,I)$ and $d(h,I)$ are different functions, which are both sensitive to I, (27) and (29) show that the current probability density functions, of negative downward flash, for conductors and structures are generally different and are both distinct from the ideal distribution to "open ground".

The widely used stroke current distributions of CIGRE and IEEE appear to have originated from a mixture of mast and transmission line conductor measurements. It becomes difficult therefore to identify true conductor or mast distributions. If we assume however that the IEEE lightning current distribution is a truly conductor distribution, the above analysis together with $r(h,I)$ and $d(h,I)$ relations obtained in ref. [4], permit the computation of the relevant distributions for masts and "open ground". The results are presented in Fig. 6 and show that for a median current of downward negative lightning of 31 kA for conductors, the corresponding values are 39 kA for masts and 23 kA for "open ground". Similar calculations with the CIGRE log-normal distribution yielded essentially the same results.

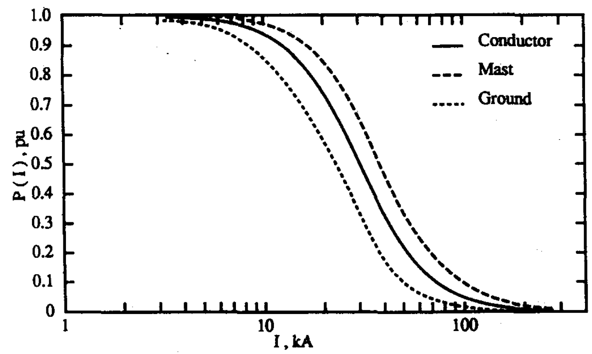

Fig. 6 Computed statistical current distributions, for negative downward lightning, for masts and "open ground" as compared to IEEE distribution, assumed valid for conductors.

Detailed calculations were undertaken, without the approximation of explicit functions in h and I only, starting from an IEEE type distribution for "open ground", with median current of 23 kA. The results showed that for conductor heights in the range 10 m-50 m, the median current varied in the very narrow range of 30.4-30.6 kA, supporting the adoption of a constant 31 kA median current for conductors.

Similar detailed calculations for masts with heights in the range 10 m-100 m, yielded median currents in the range 39.5 kA ± 5%, again indicating the current distribution is practically independent of structure height.

CONCLUSION

1. A recent physical approach to assess incidence of downward negative lightning has been extended to apply to tall masts.
2. The effect of hilly terrain on downward negative lightning was introduced by recognizing the effects of a simplified topology on induced potential and positive leader inception voltage as well as taking into account the influence of reduced air density on positive leader inception and final jump between the positive and negative leaders.
3. Two criteria must be satisfied for the occurrence

of an upward flash: the first is a complex function of ground field, structure height and topology of the terrain while the second requires a minimum ambient ground field independent of the structure height.

4. Statistical distribution functions of negative downward stroke current to conductors and masts were shown to be generally different and distinct from the ideal distribution to "open ground".

5. For conductors in the height range 10 m-50 m and masts in the range 10 m-100 m, the statistical distributions of downward negative stroke current are practically independent of height.

6. Based on an assumed median current for conductors of 31 kA, under downward negative lightning, the corresponding values for "open ground" and masts were found to be 23 kA and 39 kA respectively.

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Discussion

(For a discussion of this paper, please see the discussion following the companion paper "Modeling of Lightning Incidence to Tall Structures, Part II: Application" by F. A. M. Rizk in this issue.)